EXPERIMENTAL STUDY OF THE BEHAVIOR OF THE ENERGY SPECTRUM IN A TURBULENT FLOW IN A TUBE ROTATING RELATIVE TO ITS LONGITUDINAL AXIS

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The goal of the present study is to experimentally investigate the behavior of different characteristics in a turbulent flow. Such information is useful for developing representations of the effect of vorticity on the behavior of scales, correlation functions, and spectral distributions. The unit and experimental method employed were presented in [1] and described in more detail in [2, 3].

The unit contained a feed device with one section composed of a straight circular tube having a length equal to 100 tube diameters. This ensured a developed turbulent air flow with parameters corresponding to the literature data for the investigated regimes. The outlet section of the tube – 25 diameters long – could be rotated at a speed of 3-70 rps. Tube diameter 2a = 6 cm. Most of the measurements reported here were obtained with a flow velocity of 10 m/sec on the tube axis. Air temperature during the tests was 15-18°C. The Reynolds number was calculated from the diameter of the tube and axial flow velocity and was equal to $4 \cdot 10^4$.

The measurements were made with thermoanemometric equipment made by the DISA company. The continuous signals recorded on magnetic tape were digitally analyzed on a computer. We took steps to reduce and account for errors connected with the effect of conditions at the inlet, interference, and the presence of filters, as well as errors associated with frequency and amplitude discrimination, the superposition of spectra, the finite length of the wire of the hot-wire anemometer, and the broad variation of amplitude in the spectral distribution. Here, we resorted to smoothing within frequency intervals.

We normally used standard 1- and 2-wire sensors with wires 1.25 mm long. The diameter of the tungsten wire was $5 \cdot 10^{-3}$ mm. We also used a sensor with a wire 0.25 mm long and $2.5 \cdot 10^{-3}$ mm in diameter in order to more completely account for the dissipative part of the spectrum The error connected with the finite length of the sensor wire was estimated by the method in [4]: the error was within several percent for the intensities of the pulsations and the shear stresses; with a wire length of 1.25 mm, the error for the spectral amplitudes was less than 10% up to $k^0 = k\eta = 0.1$ but became significant at $k^0 = 0.3$. With a wire 0.25 mm long, the error was less than 5% up to $k^0 = 1$. On the whole, up to $k^0 = 0.2$, the error of the amplitudes of the spectral distributions is likely to be no more than 15%.

Below, we present and analyze data on the behavior of the unidimensional spectrum for the longitudinal component of the velocity pulsations in relation to flow direction. In the case of developed turbulent flow in a stationary channel, we compared several values with the data of Laufer and Lawn. Figure 1 shows the dimensionless rate of energy dissipation over the radius of the tube $a\varepsilon_{xx}/v_*^3$. Figure 2 shows the relation for the Taylor microscale. The agreement is satisfactory and shows that the empirical data is of acceptable accuracy. We used the following notation: $k = 2\pi f/\langle V_x\rangle$ is the wave number; $\eta = (\nu^3/\varepsilon)^{1/4}$ is the Kolmogorov microscale; $\varepsilon = 15 \cdot \varepsilon_{xx}$ is the rate of dissipation of the energy of the pulsative motion; λ is Taylor' transverse coefficient; $\lambda^2 = 10\nu\langle E\rangle/\varepsilon$, $\langle E\rangle = (1/2)\langle v_i v_i\rangle$, ν is kinematic viscosity; Λ is the longitudinal integral scale; V_* is friction rate.

Figure 3 shows Moscow Physicotechnical Institute (MFTI) data on spectral distribution in the coordinates $\varphi = kE_x(k)/\langle v_x^2\rangle$ and $k\Lambda_x$ for a flow without rotation and for different positions over the radius (lines I and II are for $Re_\lambda = 26.160$, while points 1-5 are for $\bar{r} = 0$, 0.2, 0.4, 0.6, 0.8). Figure 4 shows the same results with the outlet section of the channel rotating at 8 rps, $\Pi = \langle V_\varphi\rangle_w / \langle V_x\rangle_{max} = 0.15$.

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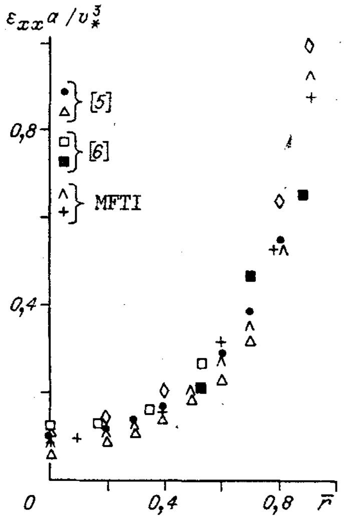

Fig. 1

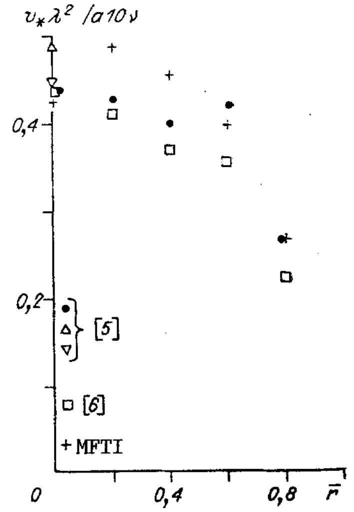

Fig. 2

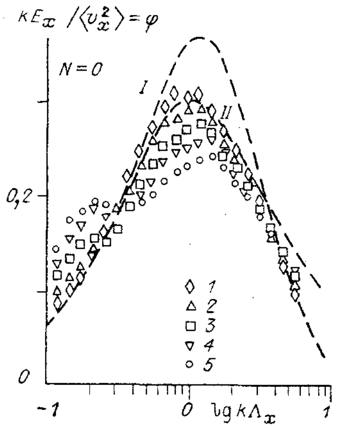

Fig. 3

With increasing distance from the tube axis, interaction with the nonuniform main flow results in distortion of the spectrum at moderate frequencies. Similar behavior is seen from the spectrum for shear stress within the same frequency range, but such distortions are absent in the spectra for the radial component of velocity fluctuation (Fig. 5, where the notation corresponds to that in Fig. 3). As a rule, the literature does not contain numerical results, while transferring data from the graphs leads to additional errors.

The behavior of the experimental spectral distributions can be compared with a generalized semi-empirical model (the EVK model) of a uniform isotropic flow [7-9] (the expression for the spectral energy distribution includes limit relations for the low-frequency interval, the inertial Kolmogrov—Obukhov frequency interval, and the dissipative frequency interval).

The unidimensional spectrum has the form [9]

$$E_x^0(y) = \int_y^\infty \frac{E^0(y_1)}{y_1}\left(1 - \frac{y^2}{y_1^2}\right) dy_1,$$

$$E^0(y) = \frac{E(k\eta)}{\nu^2}\eta = A_1\beta\left(\frac{y^2}{z^2+y^2}\right)^2 \left[(z^2+y^2)^{-5/6} + (z^2+y^2)^{-1/2}\right] \exp\left\{-A_3\left[\frac{3}{2}(z^2+y^2)^{2/3} + (z^2+y^2)\right]\right\},$$

$$\frac{1}{\beta} = 2\int_0^\infty y^2 \frac{E^0(y)}{\beta}\, dy,\ y = Q^{3/2}k^0,\ z = k_*^0 Q^{3/2},$$

$$A_1 = \alpha Q^{5/2},\ A_3 = \alpha/Q^2,\ Q = 1,\ \alpha = 1{,}65.$$

The spectral distribution curves turn out to be stratified with respect to the local Reynolds number $\mathrm{Re}_\lambda = \langle v_x^2\rangle^{1/2}\lambda/\nu$. In the distribution

$$kE_x(k)/\langle v_x^2\rangle = \varphi(\ln k\Lambda_x, \mathrm{Re}_\lambda)$$

the maximum of the curve approaches 0.27 with an increase in Re_λ. Here $\langle v_x^2\rangle$ is the intensity of the velocity fluctuations along the flow axis. The integral longitudinal scale is found numerically and can be calculated from the approximation

$$\Lambda/\eta = \mathrm{Re}_\lambda^{1/2}\left[2{,}47 + 0{,}081\left(\mathrm{Re}_\lambda - \mathrm{Re}_\lambda^{1/2}\right)\right].$$

In the flow region near the axis - where there is no intensive local interaction between the mean and fluctuation flows - the unidimensional longitudinal spectrum in the dissipative and energy-intensive frequency intervals is close to the spectral distribution for uniform isotropic flow.

Figure 6 shows results for the dependence of max φ on Re_λ for the flow in the tube: points 1-5 show MFTI data for different values of flow velocity, radial position, and number of rotations of the channel section; points 6-11 show data from [5, 6, 10-13].

The dependence of the longitudinal integral scale (connected with the value of the spectrum at the zero frequency) on Re_λ is also consistent with the behavior of the relation

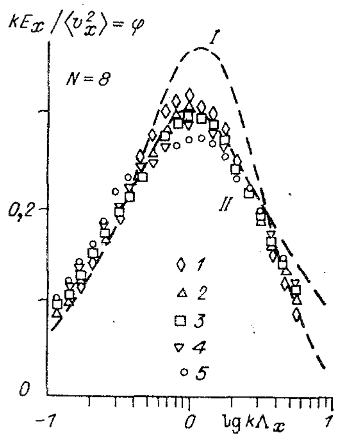

Fig. 4

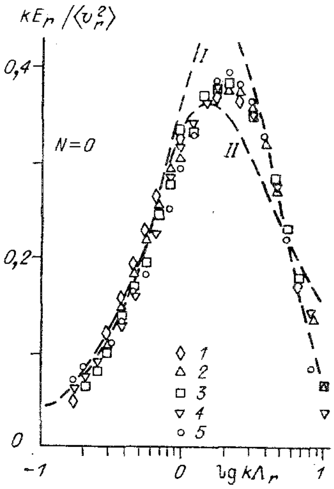

Fig. 5

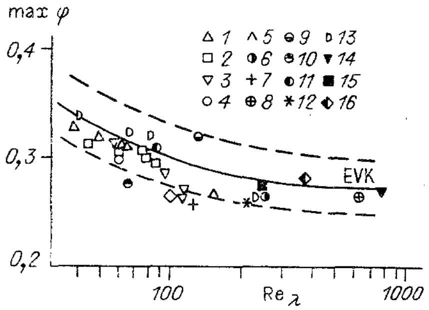

Fig. 6

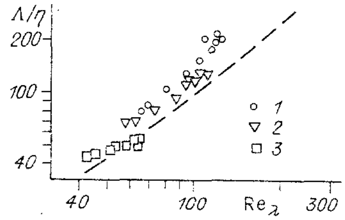

Fig. 7

for the generalized spectrum model. We were not able to precisely determine the error of the experimental measurement of the spectrum at the zero frequency, since we could not eliminate the effect of noise. This relation is shown in Fig. 7: MFTI data for different numbers of channel rotations, radial positions, and velocities (points 1-3 are for N = 0, 8, and 32). The lines in all of the figures give the relations for the generalized spectrum model (EVK model).

The maximum value of φ is a function of Re_λ which approaches the limit quite rapidly. Beginning with the value ~15, the position of this maximum as a function of the dimensionless wave number $k\Lambda$ is nearly independent of Re_λ. Here, the wave number is normalized for the integral scale calculated from an approximate expression. The distorting effect of flow nonuniformity becomes more evident with increasing distance from the tube axis. Generally speaking, this effect is small. It is true, however, that the range of variation of Re_λ was also small here.

The validity of the proposition that the local value of Re_λ has the main effect on the spectral distribution should be independent of the type of flow. Figure 6 also shows data for several flows: points 12 – plane channel [14]; 13 – wake behind a sphere, wake behind a streamlined body, and a non-impulsive wake [15]; 14 – axisymmetric jet (on the axis) [16]; 15, 16 – axis of a jet [17]. With a scatter of ±10-15%, all of the flows are described by a universal relation. The conservative behavior of the spectral distribution was noted in [15] for the flows studied in that investigation.

With normalization of the wave numbers with respect to their values on the integral scale, the spectral distributions of the intensities for the radial and azimuthal components of the velocity fluctuations also have a universal form.

The manifestation of vorticity, connected with rotation of the channel relative to the long axis of the flow, leads to a decrease in shear stress. There is also a decrease in the local effect of the mean field on the fluctuation field, with retention of the general nonlocal character of the interaction. Up to a relative radius of 0.8, the spectral energy distribution approaches the universal distribution for the generalized model. This is evident from the relations in Fig. 4.

TABLE 1

Π	$\bar{r}$	$\langle V_x \rangle$	$\langle V_\varphi \rangle$	$\langle v_x^2 \rangle$	$\langle v_r^2 \rangle$	$\langle v_\varphi^2 \rangle$	$\langle v_r v_x \rangle$	λ	η	Re_λ
		m/sec		m^2/sec^2				cm		
	0	10,02	0	0,123	0,083	0,083	0	0,276	0,0175	65
	0,2	9,82	0	0,169	0,094	0,102	0,036	0,294	0,0166	81
0	0,4	9,38	0	0,274	0,124	0,149	0,072	0,301	0,0149	106
	0,6	8,72	0	0,403	0,159	0,205	0,108	0,284	0,0131	120
	0,8	7,76	0	0,554	0,187	0,262	0,143	0,242	0,0112	120
	0	10,04	0	0,098	0,070	0,070	0	0,283	0,0186	60
	0,2	9,84	0,04	0,142	0,082	0,089	0,016	0,290	0,0172	73
0,15	0,4	9,34	0,11	0,241	0,111	0,139	0,064	0,288	0,0151	95
	0,6	8,58	0,25	0,373	0,150	0,207	0,101	0,264	0,0129	108
	0,8	7,48	0,54	0,515	0,177	0,268	0,130	0,222	0,0109	106
	0	10,08	0	0,034	0,027	0,027	0	0,343	0,0266	43
	0,2	9,94	0,02	0,048	0,029	0,032	0,009	0,352	0,0250	51
0,60	0,4	9,62	0,14	0,082	0,038	0,052	0,019	0,325	0,0209	62
	0,6	8,92	0,51	0,144	0,055	0,097	0,032	0,245	0,0158	62
	0,8	7,50	1,51	0,249	0,079	0,190	0,047	0,164	0,0113	55

Analysis of the results we have obtained shows that there is a sufficiently universal representation for spectral distributions corresponding to second-order moments for velocity fluctuations. This opens up possibilities for developing an approximate method of calculating the spectral characteristics in developed turbulent flow.

The Kolmogorov microscale and the rate of energy dissipation were measured directly by means of a sensor with a wire having a length of 0.25 mm. The value of ε is 90% at $k^0 = 0.7$, while it is about 30% at $k^0 = 0.1$. The distortion of the amplitudes of the spectrum is already substantial at $k^0 \sim 0.2$-0.3 with a wire length of 1.25 mm. It can be seen from the relations of the generalized model that $M = \max(k^0)^2 E_x^0 = 0.24$ and $k_m^0 \simeq 0.1$, regardless of the Reynolds number (beginning at $Re_\lambda = 40$). It turns out that the position of the maximum on the dissipative spectrum can be made to agree with its dimensionless value when the latter is obtained by choosing the Kolmogorov microscale as the single parameter. The two values differ only slightly in the neighborhood of the maximum for a wire length of 1.25 mm. Values of the microscale calculated in the above manner differ 10-15% from values found directly from measurements of ε for the Laufer, Lawn, and MFTI data. The same approach can be used to determine η and ε. In quantities of the type $(Re_\lambda)_x$, Λ_x, the subscript x indicates use of the intensity of velocity fluctuations in the corresponding direction.

Table 1 shows data characterizing the flow.

LITERATURE CITED

1. P. G. Zaets, A. T. Onufriev, M. I. Pilipchuk, et al., "Use of a thermoanemometric complex in a unit with a computer to measure the turbulence characteristics of vortical flows," in: Physical Methods of Studying Transparent Inhomogeneities [in Russian], Znanie, Moscow (1986).
2. P. G. Zaets, Experimental Study of the Turbulence Spectrum in a Flow in a Rotating Tube, Author's Abstract of Physical-Mathematical Candidate Dissertation, MFTI, Moscow (1986).
3. N. A. Safarov, Behavior of the Parameters of Developed Turbulent Flow in a Straight Cylindrical Channel Rotating Relative to the Longitudinal Axis, Author's Abstract of Physical-Mathematical Sciences Candidate Dissertation, MFTI, Moscow (1986).
4. J. C. Wyngard, "Measurement of small-scale turbulence with hot wires," J. Phys. E, Sci. Instrum., 1, Ser. 2, 1105 (1968).
5. J. Laufer, "Structure of turbulence in fully developed pipe flow," NACA Tech. Rept. (Washington), No. 1174 (1954).
6. C. J. Lawn, "Determination of the rate of dissipation in turbulent flow," J. Fluid Mech., 48, Part 3 (1971).
7. J. H. Pao, "Structure of turbulent velocity and scalar fields at large wave numbers," Phys. Fluids, 8, No. 6 (1965).
8. A. S. Monin and A. M. Yaglom, Statistical Fluid Mechanics [in Russian], Parts 1-2, Nauka, Moscow (1965).
9. R. J. Driscall and K. A. Kennedy, "A model for the turbulent energy spectrum," Phys. Fluids, 26, No. 5 (1983).

10. R. P. Patel, "A note on fully developed turbulent flow down a circular pipe," Aeronaut. J., 78, No. 757 (1974).
11. K. Bremhorst and K. J. Bullock, "Spectral measurements of temperature and longitudinal velocity fluctuations in fully developed pipe flow," Int. J. Heat Mass Transfer, 13, 1313 (1972).
12. K. Bremhorst and T. B. Walker, "Spectral measurements of turbulent momentum transfer in fully developed pipe flow," J. Fluid Mech., 61, Part 1 (1973).
13. W. R. Morrison and R. E. Kronauer, "Structural similarity for fully developed turbulence in smooth tubes," ibid., 39, Part 1 (1969).
14. G. Cont-Bello, Turbulent Flow in a Channel with Parallel Walls [Russian translation], Mir, Moscow (1968).
15. V. I. Bukreev, Empirical Validation of Current Representations on the Turbulent Motion of an Incompressible Fluid, Author's Abstract of Physical-Mathematical Sciences Doctoral Dissertation. Inst. Gidrodinamiki, Novosibirsk (1984).
16. M. M. Gibson, "Spectra of turbulence in a round jet," J. Fluid Mech., 151, No. 2 (1985).
17. S. Corrsin and M. S. Uberoi, "Spectra and diffusion in a round turbulent jet," NACA Tech. Memo (Washington), No. 2124 (1950).
18. P. G. Zaets, A. T. Onufriev, N. A. Safarov, and R. A. Safarov, "Experimental study of turbulent one-dimensional spectrum function in rotating pipe flow. Importance of the isotropic uniform turbulence model," Fifth EPS Liquid State Conf., Moscow, October 16-21, 1989. Proc., Moscow (1989).

RELATIONSHIP BETWEEN TURBULENT AND KINETIC ENERGY IN A MIXED SUBSTANCE

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A simple semi-empirical model of turbulent mixing is used to calculate the eddy kinetic energy of a mixed zone and to compare this value with the kinetic energy of the zone. The latter is determined by the effect of acceleration, which induces motion in the corresponding substance. This problem was examined in [1] on the basis of the model in [2]. Below, we use the approximate approach developed in [3, 4]. We compare our results with the results obtained in [1] and explain the differences – particularly for the case of impulsive accumulation. Data for impulsive acceleration is compared with experimental results in [5] and satisfactory agreement is established. This agreement could be improved if the method used to analyze the empirical data is chosen so as to be consistent with the theoretical method.

Formulation of the Problem. We will examine the problem of the mixing of two incompressible fluids of different densities located in a gravitational field. The direction of the field is such as to induce an instability which leads to turbulent mixing of the substances. Mikaelian [1] used the diffusion model in [2] to calculate the relation between the change in potential energy due to turbulent mixing and the kinetic energy acquired by a mixed substance as a result of acceleration.

We calculate the change in potential energy which is due to mixing of the substance within the interval $x_2 \leq x \leq x_1$:

$$\Delta\Pi = g_0\left[\int\limits_{x_2}^{0} (\rho_2 - \rho)\, x\, dx + \int\limits_{0}^{x_1} (\rho_1 - \rho)\, x\, dx\right]. \quad (1)$$

Here g_0 is acceleration; ρ is the density of the mixture; ρ_2 and ρ_1 are the densities of the light and heavy fluids; $x = 0$ is the position of the interface at the initial moment when the fluids are not yet mixed.

We used a notation different than that employed in [1] for the change in potential energy, in that E_t is taken to mean the turbulence energy due to the characteristic turbulent velocity v. Following [1] in designating the kinetic energy of the mixed substance as E_d, we can determine this quantity as

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